Using a tennis rating system to determine handicaps in amateur matches

Raghav Singal[1] **Timothy Chan**[2]

[1]IEOR, Columbia University, [2]MIE, University of Toronto

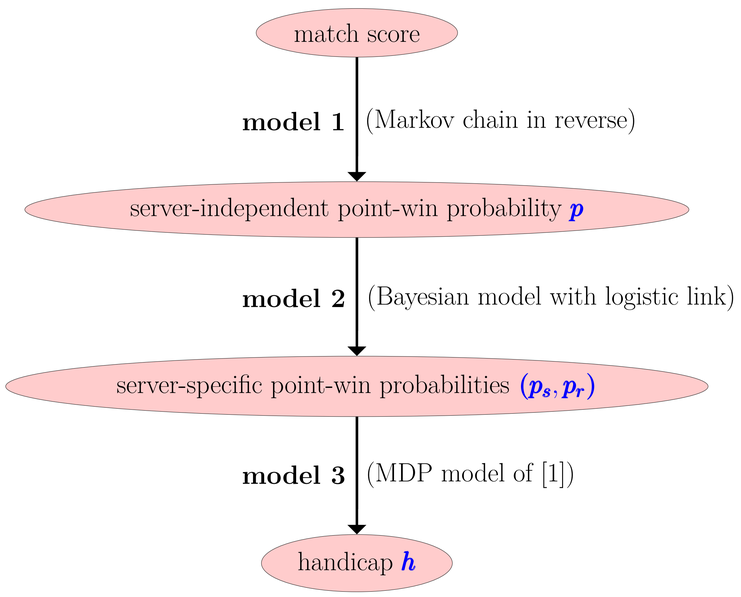
Introduction

- Handicap systems are used in sports to improve competitive balance
- Tennis does not have an official handicap system
- [1] propose a handicap system for tennis
  - The stronger player gives the weaker player h credits
  - The weaker player can use a credit at any time during the match
  - Whenever the weaker player uses one credit, she wins the point outright
  - The underlying mathematical model is a Markov Decision Process (MDP)
- [1] map server-specific point-win probabilities (p_s, p_r) to handicap h

- However, determining point-win probabilities is challenging
- On the other hand, rating systems are used widely in tennis
- **Question**: Can we map a rating difference d to a handicap h?

High-level idea of our solution

- **Final goal**: Achieve a linear mapping of the form $h = \gamma d$
- Will use a sequence of mathematical models to achieve this mapping
- For amateurs, point-level data is sparse
- Hence, we start with the match scores and the ratings differences d



- Finally, regress h against d to obtain the linear mapping $h = \gamma d$

Model 1: Markov chain

- **Goal**: Map match-score to p
- **Step 1**: Map match-score to game-win probability q
  - Suppose match-score is 3-6, 4-6
  - We estimate q as $(3+4)/(6+6+3+4) = 7/19$
- **Step 2**: Map q to p
  - Use the Markov chain model for a single game of tennis
  - Can use the absorption probability equations to solve for p given q
- **Validation**: Validated on real data from 4131 ATP / WTA matches

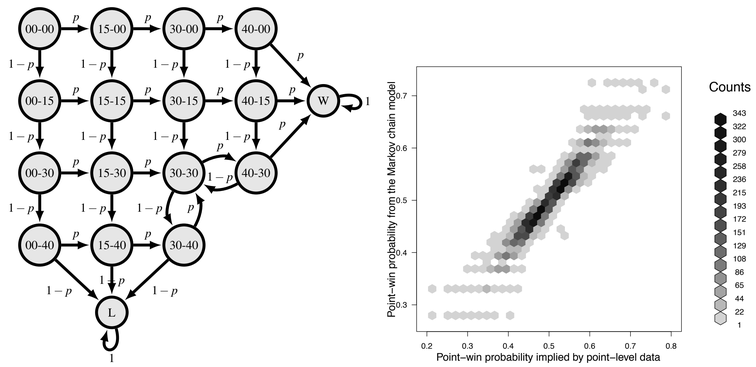


Figure : (a) Markov chain model for a single game of tennis, (b) validation

Model 2: Bayesian model with logistic link

- **Goal**: Map p to (p_s, p_r)
- **Step 1**: Map p to p_s using the following Bayesian model

$$p_s \sim \mathcal{N}_{[0,1]}\left(\frac{e^{\alpha+\beta p}}{1+e^{\alpha+\beta p}}, \sigma\right)$$

  Need to infer the three parameters α, β, and σ
- **Step 2**: Infer p_r by calibrating to the match-win probability
- **Estimation**: Estimated using real data from 2465 WTA matches

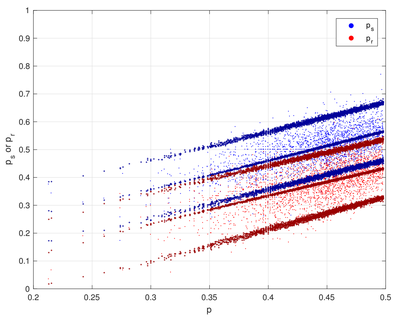


Figure : "True" data compared against Bayesian predictions

Model 3: MDP model

Map (p_s, p_r) to h using the MDP model proposed by [1]

Mapping ratings difference to handicap

- **Goal**: Map d to h
- We have already mapped match scores to handicap h
- Each match score has a corresponding ratings difference d
- **Model**: $h = \gamma d$ fitted in a Bayesian fashion

$$h \sim \mathcal{N}(\gamma d, \tau)$$

- **Result**: $h \approx 11d$ (using data from 3686 amateur matches)

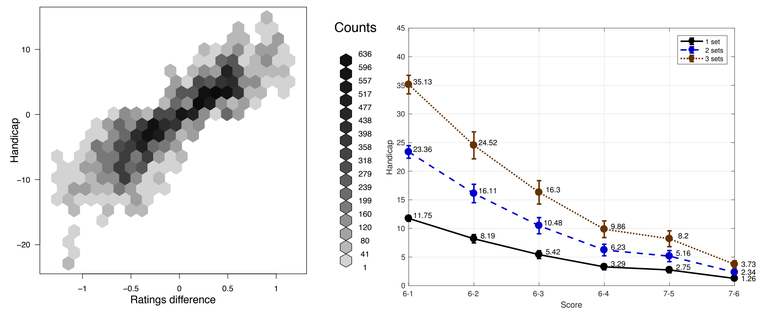


Figure : (a) Handicap against ratings difference, (b) handicaps via heuristic

Heuristic

- Some players might not have a rating but know the expected set score
- **Idea**: Map set scores to handicaps

Implementation considerations

- Multiple ways a tennis organization can implement a handicap system
- **Way 1**: Track data to estimate p and map to h using [1]
- **Way 2**: Use the heuristic presented above
- **Way 3**: Adopt a rating system and use the models presented here

Conclusion

- Developed a novel approach to map match scores to p to (p_s, p_r)
- Rigorously mapped a tennis rating system to handicaps for amateurs
- Validated models on real data from thousands of matches
- Designed an easy to remember heuristic for handicaps
- Suggested ways to implement a handicap system in real life

References

[1] Timothy CY Chan and Raghav Singal. A Markov Decision Process-based handicap system for tennis. *Journal of Quantitative Analysis in Sports*, 12(4):179–188, 2016.